A Formal Framework for e-barter based on Microeconomic Theory and Process Algebras*

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Abstract. In this paper we present a formal framework for the definition of *e-barter* architectures. By e-barter we mean the possibility of (electronically) exchanging goods without reducing transactions to money. Actually, in our setting, money can be considered just as another *good*, so that e-barter generalizes seller/buyer architectures. An advantage of e-barter systems, in contrast with most current systems, is that multilateral exchanges can be performed. Customers are first grouped into *local* markets, according mainly to their localities. Next, a higher order construction allows to compose markets, so that a *global market* takes a tree-like shape.

In order to methodically build our systems, we consider a process algebraic notation. This allows us to specify all the stages of a system (from customers to markets, markets of markets, etc). We introduce an operational semantics for our language so that exchanges of goods are formally defined. Besides, we use some concepts borrowed from microeconomic theory. Specifically, we consider utility functions (i.e. functions returning the valuation that customers/markets give to goods), exchange of goods, and equilibria.

We will show that the integration of microeconomic theory and process algebras provides two important *gains*. Firstly, it allows to avoid ambiguity in the understanding of the behavior of systems. Secondly, it gives a scheme to appropriately structure, in a bottom-up way, e-barter systems.

1 Introduction

Due to the wide implantation of internet, there has been a great proliferation of systems devoted to the (electronic) commerce of goods. However, most of these systems have very little (or not at all) theoretical foundations. During the last years there has been a great effort for setting the basis of e-commerce systems on more solid grounds, in particular, some prototypes (e.g. Kasbah [5]) have been developed in academic environments. In order to build these systems, it is important to make a clear structure of the different stages leading to the construction of an electronic marketplace (see e.g. [1]). Moreover, electronic entities replacing real buyers and sellers can be expressed in terms of (intelligent) agents

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([9] presents a survey on the topic). Even though a generalized incorporation of intelligent agents should produce a big step, current e-commerce systems make a very limited use of them, being their tasks mainly restricted to be so-called *shopping assistants*. That is, their main role consists in helping the client to find the *best deal* once the user has decided which item he wants to buy/sell. In some cases, they are also able to guide the user in the search for the corresponding good. So, it is hard to consider that these agents are *representing* the user in a *virtual* market. Ideally, users should indicate agents about their *preferences*. Then, agents should interact with other agents in order to get the goods that their users desire. Moreover, they should be real representatives of the users. In particular, they should be able to bargain for a good deal. In order to add these characteristics into agents, a formal framework is needed. Fortunately, *microeconomic theory* provides this theoretical basis. In particular, it allows to formally specify markets where customers own their products and they are willing to exchange them according to some preferences. Following this line, several proposals include, into the definition of electronic markets, either a notion of *utility*[1] or a notion of resources allocation (e.g. [13, 2, 3, 8, 6]). Such notions are very relevant if we want to provide our agents with some *negotiation* capacity [14, 10].

In this paper we propose that microeconomic concepts are very profitable for the description of some kind of electronic markets. We will mainly borrow three concepts: Utility function, exchange of resources, and equilibrium.[2] A *utility function* is simply a function $u : \mathbb{R}_+^n \to \mathbb{R}_+$ assigning a real value to any basket of resources (assuming that there are n different kinds of resources, baskets are represented as tuples belonging to $\mathbb{R}_+^n$). The higher the value returned by u is, the most preferred the basket is. A set of agents will be willing to *exchange* resources if none of them decreases its utility and at least one of them improves. Besides, we have reached an *equilibrium* (also called *Pareto optimum*) if no exchange is possible. That is, it is not possible to improve the situation of one agent without deteriorating another one. We apply these concepts to our notion of *e-barter*. In contrast to the usual understanding of e-commerce, e-barter does not necessarily reduces all the transactions to *money*: An exchange is made if both parts are *happy* with their new items. In particular, e-barter allows a richer structure of exchanges. Suppose a very simple circular situation where for each $1 \leq i \leq n$ we have that agent A_i owns the good a_i and desires the good $a_{(i \bmod n)+1}$. In our environment, this multi-agent transaction can be easily performed. On the contrary, it would not be so easy to perform it if these items must be first *converted* into money. Let us remark that money[3] can be considered just as another possible resource: A may be willing to change its good a by (m units of the good) money.

[1] It can be shown that any preference relation preserved under limits can be expressed as a continuous utility function.

[2] An easy introduction to microeconomic theory, covering the concepts appearing in this paper, can be found in [11].

[3] Let us remark that this is not the usual treatment of money in general equilibrium theory. It is not considered as a resource and it is simply used to set *relative* prices.

In order to create a *global market*, customers will be associated with *local markets*. These local markets will be also grouped into markets and so on. This structure will generate a *tree* of markets. Once the global market is created, the behavior will be as follows. Customers (more exactly, their corresponding agents) will be located at the leaves of the tree. They will exchange items within their local market until no more exchanges are possible. When such a situation is reached, their local market will try to negotiate within the next level market. That is, there will be a second order market making transactions in the name of their representatives. Again, exchanges will be made until no more exchanges are possible. Then, transactions will be performed at a third order market and so on until the root of the tree is reached. By doing so, we get that exchanges are made between agents that are as close as possible between them.

We have decided to use a formal tool to define our markets structure. They will be specified by using a process algebraic notation (see [4] for a good overview on the subject). Our language is inspired in `PAMR` [12]. This formalism was specially developed to deal with the specification and analysis of concurrent and distributed systems where resources play a fundamental role. Unfortunately, `PAMR` does not provide a *higher order* constructor as the one needed in e-barter systems, so we need to extend the language. That is, an easy modification of pure `PAMR` would allow us to define local markets, but there is no mechanism to combine markets. This fact complicates the formal definition of the language presented in this paper. Nevertheless, the addition of these features adds enough expressive power. Thus, our markets can be easily specified. In addition to a syntax, we define an operational semantics for our language. By doing so, every stage of the creation of an e-barter system may be formally specified, avoiding ambiguities and providing a clear structure of the system. Moreover, a designer of e-barter systems does not need to go through all the semantic machinery. It is enough to understand how the syntax of our language works.

The rest of the paper is structured as follows. In Section 2 we introduce some auxiliary notation. Section 3 gives an informal description of the behavior of e-barter systems. In Section 4 we present a formalization of all the concepts appearing in Section 3. Finally, in Section 5 we present our conclusions and some lines for future work.

2 Preliminaries

In this section we introduce some concepts that we will use during the rest of this paper. Specifically, we present the notions of utility function and we explain how our operational rules work. First we present some mathematical notation.

Definition 1. We consider $\mathbb{R}_+ = \{x \in \mathbb{R} | x \geq 0\}$. We will usually denote *vectors* in $\mathbb{R}^n$ (for $n \geq 2$) by $\bar{x}, \bar{y}, \ldots$. Given $\bar{x} \in \mathbb{R}^n$, x_i denotes its *i-th* component. We extend to vectors some usual arithmetic operations. Let $\bar{x}, \bar{y} \in \mathbb{R}^m$. We define $\bar{x} + \bar{y} = (x_1 + y_1, \ldots, x_n + y_n)$, and $\bar{x} \leq \bar{y}$, if for any $1 \leq i \leq n$ we have $x_i \leq y_i$.

We will usually denote *matrices* in $A^{n \times m}$ (for $n, m \geq 2$, and a set A) by calligraphic letters $\mathcal{E}, \mathcal{E}_1 \ldots$. □

The relevant characteristics of the customers of an e-barter system will be their *baskets of resources* (indicating the items that they own) and their *utility functions* (indicating preference among different baskets of resources).

Definition 2. Let us suppose that there are $m > 0$ different kinds of resources. *Baskets of resources* are defined as vectors $\bar{x} \in \mathbb{R}_+^m$. A *utility function* is any function $u : \mathbb{R}_+^m \longrightarrow \mathbb{R}$. □

In microeconomic theory, there are some restrictions that are usually imposed on utility functions (mainly strict monotonicity, convexity, and continuity). Intuitively, given a utility function u, we have that $u(\bar{x}) < u(\bar{y})$ means that the basket $\bar{y}$ is preferred to $\bar{x}$.

Finally, our operational rules will be defined as usual deduction rules. A rule

$$\frac{\text{Premise}_1 \ \wedge \ \text{Premise}_2 \ \wedge \ \dots \ \wedge \ \text{Premise}_n}{\text{Conclusion}}$$

indicates that if all of the premises hold, then we can deduce the conclusion. Premises indicate individual behavior of components of a system, while conclusions indicate how the system behaves according to individual performances.

3 An Informal Presentation of e-barter Systems

In this section we present how e-barter systems are organized (in the next section we show how they can be constructed by using our methodology). First, customers are *represented* by (electronic) agents.[4] Agents are provided with two data: The *basket of resources* that the customer is willing to exchange and a *utility function*. The utility function relates the preference that a customer has for the owned items with respect to the corresponding preference for desired items. Let us remark that, in most cases, utility functions will take a very simple form. For example, indicating that a customer C is willing to exchange the item a by the items b and c. Nevertheless, there exists several proposals showing how agents can be trained on the user preferences (see e.g. [14, 6]). Finally, let us remark that a customer has always the possibility of changing both his utility function and his basket of resources. Once the agent is notified of these changes, it will change its negotiation strategy. Besides, once an agent has reached a (possibly multilateral) deal, it must be notified to the customer. If all the customers give their approval, the deal will be effectively performed.

>From now on we concentrate on the behavior of the different electronic entities. The behavior of an e-barter system works according to the following algorithm:

1. Each agent generates the barters that its customer would be willing to perform (according to the corresponding basket of resources and utility functions).

[4] In terms of [15], our agents present as information attitude belief (vs. knowledge), while as pro-attitudes, commitment and choice (vs. intention and obligation).

2. Agents exchange goods inside their local market. A multilateral exchange will be made if (at least) one of the involved agents improves its utility and none of them decreases its utility. This is repeated until no more exchanges are possible. In this case, we say that the local market is *saturated.* In microeconomic terms, this situation is usually called equilibrium.
3. Once a market is saturated, their agents are combined to create a new agent. The new agent will have as basket of resources the addition of the corresponding to each agent. Its utility function will encode the utilities of the combined agents. Let us remark that this new agent behaves as a representative of the combined agents. *First order* agents will be combined again into markets, according to *proximity* reasons.
4. Higher order agents trade between them until their market is saturated.
5. Once a (higher order) market is saturated, the agents start to allocate the resources in a top-down way through the tree of markets until the resources arrive to the leaves of the tree (i.e. the *original* agents). Then, they create a new agent (as indicated in step 3).
6. Once their markets are saturated, new markets are created by combining agents until there exists a unique market. Once this market is saturated, and the resources are conveniently allocated, the whole tree of agents is removed, and we start again at the first step.

The previous algorithm ensures some good properties:

- Exchanges are made between agents located as near as possible. That is, we try to minimize possible shipping costs.
- Partial equilibria are reached in each market. That is, once a market is saturated we may assure that one (of the possible) Pareto optimum distribution of resources has been found. In other words, agents belonging to a saturated market cannot improve their utility within that market without decreasing the corresponding to another agent.
- Once the last (unique) market is saturated we may assure that one (of the possible) global equilibrium has been reached. That is, no more exchanges can be performed (according to the current utility functions and available resources).

Finally, let us comment on the advantages of using partial equilibria versus global equilibria. If we would pretend to reach a global equilibrium, we should perform exchanges until no more exchanges are possible. Once all this (possibly) enormous amount of exchanges has been performed, the resources can be sent to their new owners. This would strongly delay some *trivial* transactions between not very distant agents (and their corresponding customers).

4 Formalizing Agents and Markets

In this section we provide a formal syntax and semantics for the definition of e-barter systems. Even though we use a process algebraic notation (mainly when defining the operational rules) we do not need most of the usual operators for this kind of languages (choice, restriction, etc). In fact, our constructions remind a parallel operator (although we use a different syntax).

Definition 3. A *market system* is given by the following EBNF:

$$\begin{array}{lll} MS & ::= & ms(M) \\ M & ::= & A \mid \texttt{unsat}(M,\ldots,M) \mid \sigma(M) \\ A & ::= & (S,u,\bar{x}) \\ S & ::= & [\,] \mid [A,\ldots,A] \end{array}$$

□

First, we annotate market systems with the non-terminal symbol ms to avoid ambiguity of the grammar. Intuitively, $M = (S, u, \bar{x})$ (that is, $M = A$) represents a saturated market. There are two possible situations. Either S is an empty list or not. In the first case, we have that M represents an *original* agent, that is, a direct representative of a customer (note that a single agent is trivially saturated). In the second case, if $S = [A_1, \ldots, A_n]$ we have that M represents an agent associated with the (possible higher order) agents $A_1, \ldots, A_n$ belonging to a saturated market. In both cases, $\bar{x}$ represents the total amount of resources that M is responsible for, while u is its utility function. $M = \texttt{unsat}(M_1, \ldots, M_n)$ represents an *unsaturated* market consisting of the markets $M_1, \ldots, M_n$. Let us remark that in this case, some of the submarkets may be saturated and some of them unsaturated. Once all the markets of the system are saturated, the whole system is turned again into unsaturated. The term $\sigma(M)$ will represent that such operation must be performed on M. Before we introduce the operational semantics, we present an example showing how an e-barter system may be constructed. In this example we will also (informally) introduce operational transitions.

Example 1. Let us consider a total of six agents $A_i = ([\,], \overline{x_i}, u_i)$, for $1 \leq i \leq 6$. We suppose that these agents are grouped into three different markets (see Figure 1). Initially, these markets are unsaturated (unsaturated markets are represented by a single square in the figure), so we make the following definitions:

$$M_1 = \texttt{unsat}(A_1, A_2) \qquad M_2 = \texttt{unsat}(A_3, A_4) \qquad M_3 = \texttt{unsat}(A_5, A_6)$$

Suppose that the first two markets are linked, and the resulting market is also linked with the remaining M_3. We should add the following definitions:

$$M_4 = \texttt{unsat}(M_1, M_2) \qquad M_5 = \texttt{unsat}(M_4, M_3)$$

Finally, the global market is defined as $M = ms(M_5)$.

Following the philosophy explained in the previous section, transactions will be made within a market only between saturated submarkets (saturated markets are represented by doble squares in the figure). So, only M_1, M_2, and M_3 are allowed to perform transactions (note that original agents are trivially saturated).

We will denote exchange of resources by $\rightsquigarrow$. Suppose that after some exchanges, M_1 becomes saturated. That is, there exists a sequence of exchanges

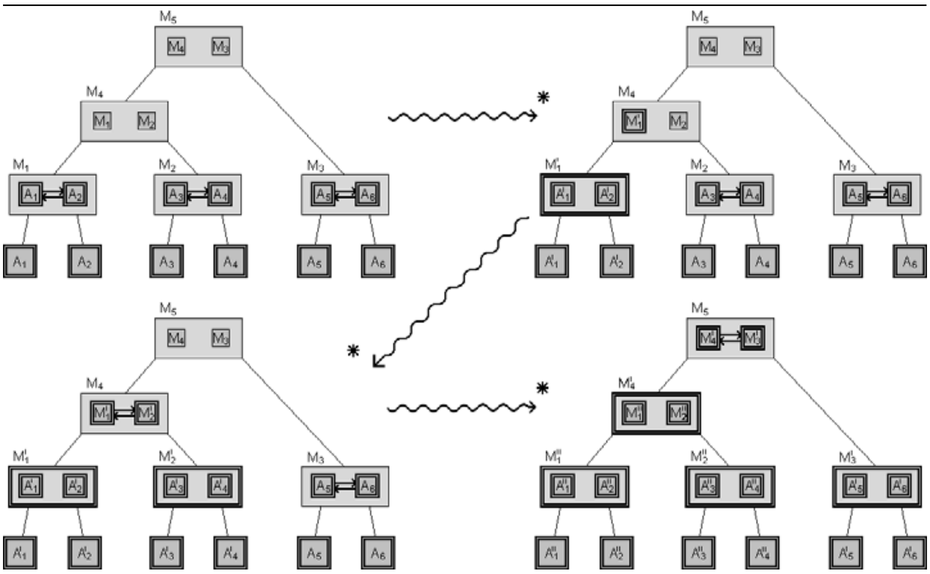

Fig. 1. Example of the evolution of a market system.

$M_1 \rightsquigarrow M_1^1 \rightsquigarrow M_1^2 \cdots \rightsquigarrow M_1^n = M_1'$ such that $M_1' \not\rightsquigarrow$. In this case, the market grouping the first two agents should be labeled as saturated. So, the agents effectively perform all the achieved transactions becoming A_1' and A_2'. Note that agents will (possibly) change their initial distributions, but the total amount of resources belonging to the local market remains constant. Then, the first market will be turned into $([A_1', A_2'], f(u_1, u_2), \overline{x_1} + \overline{x_2})$, where f is a function combining utility functions (such a function will be formally defined). In parallel, M_2 will have a similar behavior.

Once both M_1 and M_2 are saturated, the transactions between them will be allowed. Note that these transactions (inside the market M_4) will be performed according to the new utility functions, $f(u_1, u_2)$ and $f(u_3, u_4)$ respectively, and to the new baskets of resources, $\overline{x_1} + \overline{x_2}$ and $\overline{x_3} + \overline{x_4}$ respectively.

The process will iterate after M_5 gets saturated. Finally, we will have a market as $\sigma(M_5')$. Then, the global market is structurally *reset*. $\square$

In order to simplify forthcoming operational rules we introduce the following notation to deal with utility functions. Utility functions associated with original agents (that is, $A = ([\,], u, \bar{x})$) will behave as explained in Definition 2, that is $u(\bar{z})$ indicates the relative preference shown by A towards the basket of resources $\bar{z}$. Nevertheless, if $A = ([A_1, \ldots, A_n], u, \bar{x})$ then we will consider that in addition to its usual meaning, the utility function also keeps track of how a basket of resources is distributed among the (possible higher order) agents $A_1, \ldots, A_n$. That is, $u(\bar{z}) = (r, \overline{z_1}, \ldots, \overline{z_n})$, where r still represents the utility, while $\sum \overline{z_i} = \bar{z}$ and $\overline{z_i}$ denotes the portion of the basket $\bar{z}$ assigned to A_i. Overloading the notation, if we simply write $u(\bar{z})$ we are referring to the first component of the tuple, while $u(\bar{z}).i$ denotes the $(i+1)$-*th* component of the tuple.

In the next definition we present the *anchor case* of our operational semantics. In order to perform complex exchanges, agents should first indicate the barters they are willing to accept.

Definition 4. Let $A = (S, u, \bar{x})$ be a saturated market. The *exchanges* the agent A would perform are given by the following operational rules:

$$\frac{u(\bar{x}+\bar{y}) \geq u(\bar{x}) \ \wedge \ (\bar{x}+\bar{y}) \geq \bar{0}}{(S,u,\bar{x}) \xrightarrow{\bar{y}} (S,u,\bar{x}+\bar{y})} \qquad \frac{u(\bar{x}+\bar{y}) > u(\bar{x}) \ \wedge \ (\bar{x}+\bar{y}) \geq \bar{0}}{(S,u,\bar{x}) \overset{\bar{y}}{\longmapsto} (S,u,\bar{x}+\bar{y})}$$

where $\bar{y} \in \mathbb{R}^n$, being n the different kinds of available resources. □

Let us remark that in the previous definition, $\bar{y}$ may have negative components. Actually, these tuples will contain the barters offered by the agent. For example, if $\bar{y} = (1, -1, 0, 1)$ fulfills the premise, then the agent would accept a barter where it is offered one unit of the first and fourth goods in exchange of a unit of the second good. Regarding the rules, the first premise simply indicates that the agent would not decrease (resp. would increase) its utility. The second premise indicates that the agent does not run into *red numbers*, that is, an agent cannot offer a quantity of an item if it does not own enough. For example, if we consider the previous situation, we would have a transition as $A \xrightarrow{\bar{y}} A'$. Moreover, if we consider $\bar{y} = (1, -1, 0, 2)$ then we would have both[5] the transitions as $A \xrightarrow{\bar{y}} A'$ and $A \overset{\bar{y}}{\longmapsto} A'$. Let us note that $\longrightarrow$ denotes that the market does not worsen, meanwhile transition $\longmapsto$ denotes that the market does improve. Next we show how these proposals are combined.

Definition 5. Let $M = \texttt{unsat}(M_1, \ldots, M_n)$. Let $I = \{s_1, \ldots, s_r\} \subseteq \{1, \ldots, n\}$ be a set of indexes denoting the saturated markets belonging to M (that is, for any $i \in I$ we have $M_i = (S_i, u_i, \overline{x_i})$). We say that the matrix $\mathcal{E} \in (\mathbb{R}_+^m)^{n \times n}$ is a *valid exchange matrix for* M, denoted by $\texttt{valid}(M, \mathcal{E})$, if for any $1 \leq i \leq n$ we have $\sum_j \mathcal{E}_{ij} \leq \overline{x_i}$, $\mathcal{E}_{ii} = \bar{0}$, and $\forall 1 \leq k \leq n$ such that $k \notin I$, $\mathcal{E}_{ki} = \bar{0}$ and $\mathcal{E}_{ik} = \bar{0}$. □

First, let us remark that the notion of *valid* matrix is considered only in the context of unsaturated markets: If a market is saturated then no more exchanges can be performed. Second, only saturated markets belonging to an unsaturated market may perform exchanges among them. This restriction is imposed in order to give priority to transactions performed by *closer* agents belonging to unsaturated submarkets. Regarding the definition of *valid* matrix, let us note that matrixes $\mathcal{E}$ have as components baskets of resources (that is, elements belonging

[5] We are assuming that all the items are *goods*. Nevertheless, agents could also trade *bads*. For example, a customer would be willing to give an apple pie if he *receives* minus s brown leaves in his garden. However, bads are usually not considered in microeconomic theory, as they can be easily turned into goods: Instead of considering the amount of leaves, one may consider the absence of them.

to $\mathbb{R}_+^m$). $\mathcal{E}_{ij}$ represents the basket of resources that the market M_i would give to M_j. So, the condition $\sum_j \mathcal{E}_{ij} \leq \overline{x_i}$ indicates that the total amount of resources given by market M_i must be less than or equal to the basket of resources owned by that market. Finally, let us comment that an exchange does not need to include all of the saturated markets. For example, if we have an exchange where only r' markets participate, then the rows and columns corresponding to the remaining $r - r'$ saturated markets will be filled with $\bar{0}$, as they are for the unsaturated markets.

Next we introduce the rules defining the exchange of resources. Intuitively, if we have a valid exchange matrix, where (at least) one of the involved agents improves and no one worsens, then the corresponding exchange will be performed.

Definition 6. Let $M = \mathtt{unsat}(M_1, \ldots, M_n)$. Let $I = \{s_1, \ldots, s_r\} \subseteq \{1, \ldots, n\}$ be a set of indexes denoting the saturated markets belonging to M (that is, for any $i \in I$ we have $M_i = (S_i, u_i, \overline{x_i})$). The operational transitions denoting exchange of resources that M may perform are given by the rule:

$$\frac{\exists\, k \in I : M_k \overset{\overline{y_k}}{\longmapsto} M'_k \ \wedge\ \forall\, i \in I, M_i \overset{\overline{y_i}}{\longrightarrow} M'_i \ \wedge\ \mathtt{valid}(M, \mathcal{E})}{M \overset{\mathcal{E}}{\rightsquigarrow} \mathtt{unsat}(M'_1, \ldots, M'_n)} \quad \left[M'_i = \begin{cases} M_i & i \notin I \\ (S_i, u_i, \overline{x_i} + \overline{y_i}) & \text{otherwise} \end{cases} \right]$$

where $\overline{y_i} = \sum_j \mathcal{E}_{ji} - \sum_j \mathcal{E}_{ij}$ and $\mathcal{E} \in (\mathbb{R}_+^m)^{n \times n}$. We say that M is a *local equilibrium*, denoted by $M \not\rightsquigarrow$, if there do not exist $M', \mathcal{E}$ such that $M \overset{\mathcal{E}}{\rightsquigarrow} M'$. □

The previous operational rule is applied under the same conditions appearing in the definition of a valid exchange matrix: It is applied to unsaturated markets and the exchange is made among a subset of the saturated submarkets. The premises indicate that, at least, an unsaturated market will improve after the exchange and that no one deteriorates. Let us remind that, in general, a market may generate both $M_i \overset{\bar{y}}{\longrightarrow} M'_i$ and $M_i \overset{\bar{y}}{\longmapsto} M'_i$. So, the previous rule also considers situations where more than a market improves (we only require that, at least, one improves). Besides, let us remark that $M_i \overset{\bar{0}}{\longrightarrow} M'_i$ always holds. So, a market not involved in the current exchange does not disallow the exchange. Regarding the conclusion, submarkets belonging to M are modified according to the corresponding exchange matrix, while unsaturated submarkets do not change. We need to consider two more exchanging rules.

$$\frac{M_k \overset{\mathcal{E}}{\rightsquigarrow} M'_k}{\mathtt{unsat}(M_1, \ldots, M_k, \ldots, M_n) \overset{\mathcal{E}}{\rightsquigarrow} \mathtt{unsat}(M_1, \ldots, M'_k, \ldots, M_n)} \qquad \frac{M \overset{\mathcal{E}}{\rightsquigarrow} M'}{ms(M) \overset{\mathcal{E}}{\rightsquigarrow} ms(M')}$$

The first one indicates that if an unsaturated submarket produces an exchange, then the market must take that situation into account. The second rule reflects modifications in the environment of the constructor ms.

If a market reaches an equilibrium, we have that it has become saturated. In this case, we need to modify the attribute of the market, replacing a term $\mathtt{unsat}(M_1, \ldots, M_n)$ by an adequate term as $(S, u, \bar{x})$. In addition, resources are recursively moved from the corresponding agents to the leaves of the tree (indicating original agents). Let us remark that a necessary condition for a market to be saturated is that all its submarkets are also saturated. The following rule uses two auxiliary notions that will be formally presented in the forthcoming Definition 8.

Definition 7. Let $M = \mathtt{unsat}(M_1, \ldots, M_n)$ be a market, where we have that $M_i = (S_i, u_i, \bar{x}_i)$ for any $1 \leq i \leq n$. The following rule modifies the market from unsaturated to saturated:

$$\frac{M \not\rightsquigarrow}{M \rightsquigarrow ([M'_1, \ldots, M'_n], u, \sum \overline{x_i})}$$

where $u = \mathtt{CreateUtility}(u_1, \ldots, u_n, \overline{x_1}, \ldots, \overline{x_n})$ and for any $1 \leq i \leq n$ we have that $M'_i = (S'_i, u_i, \overline{x_i})$ and $S'_i = \mathtt{Deliver}(S_i, u_i, \overline{x_i})$. □

Let us remark that in this rule we do not label the transition $\rightsquigarrow$. These transitions play a role similar to internal transitions in classical process algebras. We need to add two more rules, as in the previous case, to record this transformation in the context of different constructors:

$$\frac{M_k \rightsquigarrow M'_k}{\mathtt{unsat}(M_1, \ldots, M_k, \ldots, M_n) \rightsquigarrow \mathtt{unsat}(M_1, \ldots, M'_k, \ldots, M_n)} \qquad \frac{M \rightsquigarrow M'}{ms(M) \rightsquigarrow ms(M')}$$

Next we present the pending auxiliary functions. Intuitively, the function $\mathtt{Deliver}(S, u, \bar{x})$ distributes the basket of resources $\bar{x}$ among the original agents appearing in the leaves of the tree S. This distribution takes into account utility functions of the agents, as well as the quantities of resources contributed by each agent. On the other hand, $\mathtt{CreateUtility}(u_1, \ldots, u_n, \overline{x_1}, \ldots, \overline{x_n})$ computes a combined utility function from the ones provided as arguments, so that it is possible to negotiate for maximizing the overall profit of the represented agents. Let us remind that, in this section, utility functions associated with higher order agents do not only reveal preference. In addition, they also take into account how resources will be distributed among agents. Thus, if we are considering an agent representing n agents, a new utility function returning a tuple with $n + 1$ components will be created. The first component (the value of the utility function) will return the worst utility (0) if any of the represented agents worsens. In this way, it is guaranteed that the market does not make any exchange which deteriorates any of its clients.

Definition 8. Let $A = (S, u, \bar{x})$ be an agent. The *allocation* of the basket of resources $\bar{x}$ among the agents belonging to S, with respect to the utility function u, denoted by $\mathtt{Deliver}(S, u, \bar{x})$, is recursively defined as:

$$\mathtt{Deliver}(S, u, \bar{x}) = \begin{cases} [\,] & \text{if } S = [\,] \\ [M'_1, \ldots, M'_n] & \text{if } S = [M_1, \ldots, M_n] \end{cases}$$

where for any $1 \leq i \leq n$ we have that $M_i = (S_i, u_i, \overline{x_i})$, $M'_i = (S'_i, u_i, u(\bar{x}).i)$, and $S'_i = \mathtt{Deliver}(S_i, u_i, u(\bar{x}).i)$.

Given n pairs $(u_i, \overline{x_i})$ we define the *utility function* defined from the utility functions $u_1, \ldots, u_n$ with respect to the *initial* baskets of resources $\overline{x_1}, \ldots, \overline{x_n}$, denoted by $\texttt{CreateUtility}(u_1, \ldots, u_n, \overline{x_1}, \ldots, \overline{x_n})$, as:

$$\texttt{CreateUtility}(u_1, \ldots, u_n, \overline{x_1}, \ldots, \overline{x_n}) = u_{market}$$

where $u_{market}(\overline{x}) = max(\{(r, \overline{x_1'}, \ldots, \overline{x_n'}) | r = \sum_{1 \leq i \leq n} u_i(\overline{x_i'}) \wedge \sum_{1 \leq i \leq n} \overline{x_i'} = \overline{x} \wedge \bigwedge_{1 \leq i \leq n} u(\overline{x_i'}) \geq u(\overline{x_i})\})$, maximizing over the first argument (representing the *utility*), and supposing $\max(\emptyset) = (0, \bar{0}, \ldots, \bar{0})$. □

Next, to define how a market evolves, we compose sequences of transitions.

Definition 9. We say that a market M *evolves into* a market M', and we write $M \rightsquigarrow^* M'$, if there exist markets $M_1, \ldots, M_{n-1}$ such that

$$M \overset{a_1}{\rightsquigarrow} M_1 \overset{a_2}{\rightsquigarrow} M_2 \overset{a_3}{\rightsquigarrow} \cdots M_{n-1} \overset{a_n}{\rightsquigarrow} M'$$

where for any $1 \leq i \leq n$ we have a_i is an empty label or an exchange matrix. □

Finally, we provide a mechanism to reset a global market. If the root of the tree becomes a saturated market, then the whole tree of markets is created again. This is done by considering the following five rules:

$$\frac{}{ms((S, u, \bar{x})) \hookrightarrow ms(\sigma((S, u, \bar{x})))} \qquad \frac{}{\sigma(([\,], u, \bar{x})) \hookrightarrow ([\,], u, \bar{x})}$$

$$\frac{S = [A_1, \ldots, A_n]}{\sigma((S, u, \bar{x})) \hookrightarrow \texttt{unsat}(\sigma(A_1), \ldots, \sigma(A_n))}$$

$$\frac{M_k \hookrightarrow M_k'}{\texttt{unsat}(M_1, \ldots, M_k, \ldots, M_n) \hookrightarrow \texttt{unsat}(M_1, \ldots, M_k', \ldots, M_n)} \qquad \frac{M \hookrightarrow M'}{ms(M) \hookrightarrow ms(M')}$$

We finish this section by providing a result showing that the last saturated market represents one of the possible Pareto optimums for the whole set of original agents. The proof is immediate.

Theorem 1. Let M, M' be markets and A be an agent such that

$$M \rightsquigarrow^* M' \hookrightarrow ms(\sigma(A))$$

The distribution of resources provided by $\sigma(A)$ represents a Pareto optimum with respect to the original agents belonging to M. □

5 Conclusions and Future Work

We have presented a formal framework for the definition of *e-barter* architectures. In contrast with most current systems, by using e-barter, multilateral exchanges can be performed. The use of a hierarchical structure of markets allows local markets to evolve in parallel, but preserving the property that the global market still reaches Pareto equilibria. By doing so,the efficiency is notably increased, as most of the exchanges are done in (near) local markets. Let us remark that the integration of microeconomic and process algebra has provided two important advantages. Firstly, it has avoided ambiguity in the understanding of the behavior of systems. Secondly, it has allowed to appropriately structure, in a bottom-up way, e-barter systems. As future work we plan to study the behavior of markets where customers pay commissions when exchanging goods. Thus, it can be modelled how the markets are distorted when intermediates appear. We also plan to study alternative semantics. In particular, based on the work presented in [7] for analyzing the termination of processes, we are working on a testing semantics for characterizing markets equilibria.

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